Calculus with Mathematica

1. Abstract

The Calculus with Mathematica project is devoted to improving conceptual understanding of the calculus by making use of the Mathematica technical computing software to solve more challenging kinds of applications than are traditionally examined in undergraduate calculus.

This MUSPIN-funded pilot project has been implemented at South Carolina State University for two semesters and a summer at the level of introductory differential calculus. I will present preliminary results, including our successes and the barriers to further success and the impact on the over 50 MSET students enrolled in the course during the past year. Also presented will be our plans for implementation in the academic year 1999 – 2000 and beyond.

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2. What is Calculus with Mathematica?

Calculus with Mathematica is a strategy for teaching calculus that makes use of a technical computing software that helps students become better problem solvers by

- Learning visually (observing and analyzing data by using computer generated tables and graphs)
- Using computer generated images to assist in forming mental images or representations of mathematical processes, such as forming the inverse of a function or taking its derivative
- Learning by experimenting with examples to discover relationships, and to make and test conjectures
- Learning to work as a team

Calculus with Mathematica is certainly beneficial to the "A" student. But more importantly, it is appealing to the student that

- Wants to learn math but feels that traditional teaching has been inadequate for their needs
- Finds it difficult to learn by reading printed math textbooks
- Likes to work with others in teams or partnerships
- Is not particularly confident in their ability to do algebra and other symbolic manipulations

Calculus with Mathematica allows for more real-life connections, more incorporation of technology, and the development of alternative forms of teaching, assessment, and learning.

3. Participants

Students enrolled in *Calculus with Mathematica* are, for the most part, sophomores majoring in one of the following four programs: Engineering Technology, Biology, Mathematics, Computer Science. They are students who randomly selected the *Calculus with Mathematica* section of Calculus I (There are two other sections of Calculus I that are taught using the traditional method.). Other than major, students enrolled in the *Calculus with Mathematica* section fit no particular profile and can be thought of as *average* students of calculus.

4. Description of the Teaching Method

The instructor of The Calculus with Mathematica Course holds the philosophy that a change in pedagogical style is particularly important. Students should be active learners and should learn to think autonomously. Students should work together in small cooperative learning groups. Cooperative learning groups, he feels, replace competition with cooperation, promote conversations about mathematics, and provide students of different strengths and learning styles the opportunity to contribute to the solution of problems.

Calculus with Mathematica is a three-credit course. Students meet three periods per week, fifty minutes per period. Lecture and recitations are given during the first two periods. Generally, the Mathematica Lab is held during the third period. However, depending upon the concept, the order is sometimes reversed, when introducing a new idea is enhanced by computer visualizations.

During the *Mathematica* Lab period, students work in groups of two's and three's to solve problems that are unfamiliar, but which deal with mathematically related topics that are currently being

studied in lecture. Each student, however, must submit a solution sheet that is then graded by a lab assistant. An occasional *Mathematica* Lab question is placed on regularly given quizzes/tests that cover lecture/recitation information. The *Mathematica* lab grade accounts for 15% of the student's final grade.

Many calculus courses are crammed far too full with topics, with the result that topics are given far too superficial a treatment. Instead, *Calculus with Mathematica* converges on a smaller number of topics that must be learned well. The following topics are covered in considerable detail:

- Introduction to Mathematica syntax
- Functions and their representations, including numerical, graphical and symbolic
- Limits and continuity from an intuitive and constructive point of view
- The derivative of a function as a function that gives the relative rate of change or the slope of a tangent
- Techniques of differentiation
- Fundamental theorems involving the first and second derivatives
- The Mean Value Theorem in an intuitively clear form: If f' > 0 on an interval, then f is increasing on that interval.
- Differentiation applied to Min/Max problems
- Differentiation applied to other applications (motion, related rates, Newton's Method)

5. Preliminary Results: Successes, Impact, Barriers

Successes, Impact

Although the final class mean for students enrolled in the *Calculus with Mathematica* section of Calculus I has not differed significantly from those of students enrolled in the sections taught using the traditional method, several observations are worth mentioning:

- The withdrawal rate in the *Calculus with Mathematica* section is less than 7% as compared to nearly 12% in the sections taught using the traditional method.
- Average daily attendance in the *Calculus with Mathematica* section is around 92% as compared to around 76% in the sections taught using the traditional method.
- Students who achieve less than a "C" in the *Calculus with Mathematica* section and must repeat Calculus I almost always choose to re-enroll in the *Calculus with Mathematica* section rather than change to a traditional section.
- Students who complete the *Calculus with Mathematica* section almost always succeed in the next course in the calculus sequence, obtaining a grade at least as high as that achieved in the *Calculus with Mathematica* course.
- Students say they learn more when they work in teams of two or three. When working alone, they say, they miss a lot of things that teams tend not to miss.
- Students say they like the personal attention. Attached to the *Calculus with Mathematica* lab is an instructor and an undergraduate assistant. To get a consultation during lab time, all a student has to do is raise a hand. Everyone gets to know each other in a way impossible in the traditional model of teaching.

Barriers

The Calculus with Mathematica section of Calculus I does have some barriers:

- The network on which the *Mathematica* software is accessed is totally dependent upon the University's network system; the University's network system is occasionally inaccessible.
- The Calculus with Mathematica section of Calculus I must utilize the same labs as are used for other computer science courses. Scheduling is sometimes problematic.
- Occasionally, being enrolled in the *Calculus with Mathematica* section of Calculus I requires additional lab time, which some students dislike.

6. Future Implementation

- Currently, the number of *Mathematica* commands used for lab assignments is small and are generally only of the basic type (Fit, Plot, Solve, /., Replace, Limit, D). In the future, we will expand the use of Mathematica's capabilities to include more graphics and pattern structures, thus enhancing students' critical thinking.
- Presently, Calculus with *Mathematica* is utilized only in introductory differential calculus. Considerations are under way to utilize *Mathematica* in other levels of the calculus sequence.